Computing Differential Modular Forms

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Let $p > 3$ be a prime number.

Let R be a complete, discrete valuation ring whose maximal ideal is generated by p and whose residue field k is algebraically closed.

Let $\phi : R \to R$ be the unique lifting of the Frobenius morphism to R.

Let $\delta : R \to R$ be the map

$$\delta(x) = \frac{\phi(x) - x^p}{p}.$$

Then δ is the unique p-derivation on R of the identity. In general

Definition 1 *A p-derivation is a set theoretic map, $\delta : A \to B$, from a ring A to an A-algebra B such that*

$$\delta(x+y) = \delta x + \delta y + C_p(x,y)$$
$$\delta(xy) = y^p \delta x + x^p \delta y + p\delta x \delta y$$

for all $x, y \in A$ where $C_p(X,Y) = \frac{X^p + Y^p - (X+Y)^p}{p}$.

Very Computable with "Calculus type" Rules!

First we must define an extension of $C_p(X, Y)$.

Definition 2 *For any* $\sum q$, *let*

$$C_p^{ext}(\sum q) = \frac{\sum q^p - (\sum q)^p}{p}$$

Note that $C_p^{ext}(X+Y) = \frac{X^p+Y^p-(X+Y)^p}{p} = C_p(X, Y)$, and so this is a very natural definition.

Lemma 1 *Let* $\delta : A \to B$ *be a* p*-derivation, let* $g = \sum q, x, y \in A$, *and let* $n > 0$ *be an integer. Then the following are true.*

1. $\delta(\sum q) = \sum \delta q + C_p^{ext}(\sum q)$
2. $\delta(-1) = 0$
3. $\delta(-x) = -\delta x$
4. $\delta(x^n) = \sum_{k=1}^{n} \binom{n}{k} p^{k-1} x^{(n-k)p} (\delta x)^k = \frac{-x^{np} + (x^p + p\delta x)^n}{p}$
5. $\delta\left(\frac{1}{x}\right) = \frac{-\delta x}{x^p(x^p + p\delta x)}$
6. $\delta\left(\frac{y}{x}\right) = \frac{x^p \delta y - y^p \delta x}{x^p(x^p + p\delta x)}$

Consider the following sequence of R-algebras

$$R \xrightarrow{f,\delta} M \xrightarrow{f^1,\delta} M^1 \xrightarrow{f^2,\delta} M^2 \longrightarrow \dots$$

where

$$\begin{aligned} M &= R[a_4, a_6, \Delta^{-1}]\hat{} \\ M^1 &= R[a_4, a_6, \delta(a_4), \delta(a_6), \Delta^{-1}]\hat{} \\ M^2 &= R[a_4, a_6, \delta(a_4), \delta(a_6), \delta^2(a_4), \delta^2(a_6), \Delta^{-1}]\hat{} \end{aligned}$$

each f^i is the natural embedding, $\Delta = -2^4(4a_4^3 + 27a_6^2)$, and δ maps

$$\begin{aligned} a_4 &\mapsto \delta(a_4) \\ a_6 &\mapsto \delta(a_6) \\ \delta(a_4) &\mapsto \delta^2(a_4) \\ \delta(a_6) &\mapsto \delta^2(a_6) \end{aligned}$$

Then if $X = \operatorname{Spec} \mathrm{M}$, the first jet space of X would be $\operatorname{Spec} \mathrm{M}^1$.

Let E be the elliptic curve over M^0 defined by the homogeneous equation

$$f(X,Y,W) = WY^2 - X^3 - a_4XW^2 - a_6W^3.$$

Let U and V be the affine open subsets of E given by the equations $f(x,y,1)$ and $f(z,1,w)$ respectively. So

$$\begin{aligned} U &= \mathrm{Spec}\, \mathrm{M}^0[\mathrm{X},\mathrm{Y}]/(\mathrm{f}(\mathrm{X},\mathrm{Y},1)) = \mathrm{Spec}\, \mathrm{M}^0[\mathrm{x},\mathrm{y}] \\ V &= \mathrm{Spec}\, \mathrm{M}^0[\mathrm{Z},\mathrm{W}]/(\mathrm{f}(\mathrm{Z},1,\mathrm{W})) = \mathrm{Spec}\, \mathrm{M}^0[\mathrm{z},\mathrm{w}] \end{aligned}$$

and on $U \cap V$

$$\begin{aligned} z &= -x/y \\ w &= -1/y. \end{aligned}$$

whence $E = U \cup V$ Next we define the first jets of U and V to be the sets

$$\begin{aligned} U^1 &= \mathrm{Spec}\, \mathrm{M}^1[\mathrm{X},\mathrm{Y},\delta\mathrm{X},\delta\mathrm{Y}]/(\mathrm{f}(\mathrm{X},\mathrm{Y},1),\delta\mathrm{f}(\mathrm{X},\mathrm{Y},1)) = \mathrm{Spec}\, \mathrm{M}^1[\mathrm{x},\mathrm{y},\delta\mathrm{x},\delta\mathrm{y}] \\ V^1 &= \mathrm{Spec}\, \mathrm{M}^1,[\mathrm{Z},\mathrm{W},\delta\mathrm{Z},\delta\mathrm{W}]/(\mathrm{f}(\mathrm{Z},1,\mathrm{W}),\delta\mathrm{f}(\mathrm{Z},1,\mathrm{W})) = \mathrm{Spec}\, \mathrm{M}^1[\mathrm{z},\mathrm{w},\delta\mathrm{z},\delta\mathrm{w}] \end{aligned}$$

Then E^1, the first jet space of E, is the gluing of U^1 and V^1 by the maps

$$\begin{aligned} z &= -x/y \\ w &= -1/y \\ \delta z &= \frac{x^p\delta y - y^p\delta x}{y^p(y^p + p\delta y)} \\ \delta w &= \frac{\delta y}{y^p(y^p + p\delta y)} \end{aligned}$$

Just as E is a group scheme, E^1 is also a group scheme. We can express this group law in terms of coordinates on either U or V.

On V, the origin is $(0,0,0,0)$. Let $P_i = (z_i, w_i, \delta z_i, \delta w_i)$. Then the inverse of P_0 is $-P_0 = (-z_0, -w_0, -\delta z_0, -\delta w_0)$. The coordinates P_3 of the sum $P_1 \oplus P_2$ are given by

$$\begin{aligned} z_3 &= -\frac{\alpha}{\mu} \\ w_3 &= -\frac{\beta}{\mu} \\ \delta z_3 &= -\frac{\mu^p \delta\alpha - \alpha^p \delta\mu}{\mu^p(\mu^p + p\delta\mu)} \\ \delta w_3 &= -\frac{\mu^p \delta\beta - \beta^p \delta\mu}{\mu^p(\mu^p + p\delta\mu)} \end{aligned}$$

where

$$\begin{aligned} \alpha &= -2w_2z_1 - w_1z_1 + 2w_1z_2 - 3a_6w_2w_1^2z_2 - a_4w_1^2z_2^2 + w_2z_2 \\ &\quad + 3w_2^2z_1a_6w_1 + w_2^2z_1^2a_4 \\ \beta &= 3w_2z_2z_1^2 + z_1w_2^2w_1a_4 - 3z_1w_1z_2^2 + w_1^2 - w_2w_1^2z_2a_4 - w_2^2 \\ \mu &= 3a_6w_2w_1(w_2 - w_1) + 3z_2z_1(z_2 - z_1) + a_4(w_2^2z_1 - w_1^2z_2) \\ &\quad + w_1 - w_2 + 2a_4w_2w_1(z_2 - z_1). \end{aligned}$$

Note: If $z_1 = z_2$, it is necessary to take the limits of the group law.

Let

$$M(R) = \{(a,b) \in R^2 | 4a^3 + 27b^2 \in R^*\}.$$

Then the set $M(R)$ is in one-to-one correspondence with the set of pairs consisting of an elliptic curve over R and an invertible 1-form.

For any $f \in M^1$ if we substitute $a, b, \delta a, \delta b$ in for $a_4, a_6, \delta a_4, \delta a_6$, then f defines a map (still denoted by f) from $M(R)$ to R. This element in M^1 is in fact uniquely determined by the map from $M(R)$ to R. Therefore we call f a δ modular function of order 1.

We define a δ-*character* of order ≤ 1 to be a group homomorphism $\chi : R^* \to R^*$ of the form $\chi = \chi_{m,n}$ where

$$\chi_{m,n}(\lambda) = \lambda^m \left(\frac{\phi(\lambda)}{\lambda^p} \right)^n .$$

A δ modular function has weight χ if for any $\lambda \in R^*$

$$f(\lambda^4 a, \lambda^6 b) = \chi(\lambda) f(a,b)$$

for all $(a,b) \in M(R)$. A δ modular form is a δ modular function with a weight.

A δ modular form is *isogeny-covariant* if for any two pairs (a, b) and $(\tilde{a}, \tilde{b})$ with an etale isogeny of degree N between the corresponding elliptic curves that pulls back $\frac{dx}{y}$ to $\frac{dx}{y}$

$$f(a, b) = N^{-k/2} f(\tilde{a}, \tilde{b})$$

where k is a constant that depends solely on the weight. Note that for $\chi = \chi_{m,n}$ the constant is $k = m + n(1 - p)$.

Theorem 2 (Buium) *For all δ-characters, χ, of order ≤ 1 there is up to multiplication by a constant in $\mathbb{Z}_p$ a unique isogeny covariant δ modular form of weight χ.*

We describe from the construction of the unique isogeny covariant differential modular form of weight $\chi_{-p-1,-1}$, called f_{jet} now to compute f_{jet}. We must

- Find two sections

  $$s_U : U^0 \otimes M^1 \to U^1$$

  and

  $$s_V : V^0 \otimes M^1 \to V^1.$$

- Find the difference of sections under the group law, $s_V - s_U$, which determines a map from $(U \cap V)^0 \otimes M^1 \to E^1$. Select the $\delta z = \zeta$ coordinate from this difference.
- Apply $\log_{\mathcal{F}_1^{\phi^1}}(\xi)$, the formal logarithm of the Frobenius twist of the formal group of the elliptic curve to $\zeta = \delta z$.
- Take the residue class of $\log_{\mathcal{F}_1^{\phi^1}}(\zeta)$

Then $f_{jet} =< \log_{\mathcal{F}_1^{\phi^1}}(\zeta), \omega >$, the residue class of $\log_{\mathcal{F}_1^{\phi^1}}(\zeta)$.

Proposition 3 *The section s_U modulo p^2 is*

$$(x, y, -A^p(P_{U,0} + pP_{U,1}), -B^p(P_{U,0} + pP_{U,1}))$$

where

$$\begin{aligned}
A &= \frac{2^4(4a_4^2 + 6x^2a_4 - 9xa_6)}{\Delta} \\
B &= \frac{2^3(9y)(2xa_4 - 3a_6)}{\Delta} \\
P_{U,0} &= -\delta a_4 x^p - \delta a_6 + C_p^{ext}(y^2 - x^3 - a_4x - a_6) \\
R_{U,0} &= C_p^{ext}(Af_x + Bf_y) + A^p(-\delta(3)x^{2p} \\
&\quad + C_p^{ext}(-3x^2 - a_4)) + B^p\delta(2)y^p \\
P_{U,1} &= -3x^pP_{U,0}^2A^{2p} + \delta a_4P_{U,0}A^p + P_{U,0}^2B^{2p} - P_{U,0}R_{U,0}.
\end{aligned}$$

Proposition 4 *The section s_V modulo p^2 is*

$$(z, w, -C^p(P_{V,0} + pP_{V,1}), -D^p(P_{V,0} + pP_{V,1}))$$

where

$$\begin{aligned}
C &= z(-\frac{3}{2}a_6w - a_4z) \\
D &= -\frac{3}{2}a_6w^2 - wa_4z + 1 \\
P_{V,0} &= -\delta a_6w^{3p} - \delta a_4z^pw^{2p} + C_p^{ext}(w - z^3 - a_4zw^2 - a_6w^3) \\
R_{V,0} &= C_p^{ext}(Cg_z + Dg_w) + C^p(-\delta(3)z^{2p} + C_p^{ext}(-3z^2 - a_4w^2)) \\
&\quad + D^p(-\delta(3)a_6^pw^{2p} - \delta(2)a_4^pz^pw^p + C_p^{ext}(1 - 3a_6w^2 - 2a_4zw)) \\
P_{V,1} &= -3z^p(P_{V,0}C^p)^2 + (2a_4^pw^p(-P_{V,0}D^p) + \delta a_4w^{2p})(P_{V,0}C^p) \\
&\quad - (a_4^pz^p + 3a_6^pw^p)(P_{V,0}D^p)^2 - (3\delta a_6w^{2p} + 2\delta a_4z^pw^p)(-P_{V,0}D^p) \\
&\quad - P_{V,0}R_{V,0}
\end{aligned}$$

Proposition 5 *The $\zeta = \delta z$ term modulo p^2 of $s_U - s_V$ is*

$$\zeta = \frac{-\delta\alpha}{(8w)^p} + p\frac{\delta\alpha\delta\mu}{(8w)^{2p}}$$

where $\delta\alpha$ modulo p^2 is

$$\begin{aligned}
\delta\alpha = \Bigg(&\Big(2(3^p)w^{5p}a_6^pB^pA^p - 2w^{3p}B^pA^p - 3w^{3p}B^{2p}z^p + 2^pw^{3p}B^pA^p + 4w^{4p}a_4^pz^pB^pA^p + w^{4p}a_4^pA^{2p}\Big)P_{U,0}^2 \\
&+ \Big(\big(2^pw^pD^pz^pB^p + 4w^{2p}D^pa_4^pz^{2p}B^p - 2(3^p)w^{4p}C^pa_6^pB^p + 4w^{2p}D^pa_4^pz^pA^p + 2(3^p)w^{3p}D^pa_6^pA^p \\
&+ 2(3^p)w^{3p}D^pa_6^pz^pB^p - 4w^{3p}B^pa_4^pz^pC^p + 2^pw^pD^pA^p - 2^pC^pB^pw^{2p}\big)PV,0 \\
&- 3^pw^{4p}\delta(a_6)A^p - \delta(2)w^{2p}A^p - 2w^{3p}\delta(a_4)z^pA^p - \delta(3)w^{4p}a_6^pA^p\Big)PU,0 \\
&+ \Big(- 3^pD^pC^pa_6^pw^{2p} + a_4^pz^{2p}D^{2p} + D^pC^p - w^{2p}a_4^pC^{2p} + 3^pz^pa_6^pw^pD^{2p}\Big)P_{V,0}^2 \\
&+ \Big(- 2w^pD^p\delta(a_4)z^{2p} - \delta(3)z^pa_6^pw^{2p}D^p - 3^pD^pz^p\delta(a_6)w^{2p} + 2w^{2p}\delta(a_4)z^pC^p \\
&+ \delta(2)C^pw^p - \delta(2)z^pD^p + 3^pw^{3p}C^p\delta(a_6) + \delta(3)w^{3p}C^pa_6^p\Big)PV,0 \\
&+ \Big(w^{2p}A^p - 2w^{3p}a_4^pz^pA^p + 2w^{2p}z^pB^p - 3^pw^{4p}a_6^pA^p - 2^pw^{2p}A^p\Big)PU,1 \\
&+ \Big(2^pC^pw^p - D^pz^p + 2w^{2p}a_4^pz^pC^p - w^pC^p - 2a_4^pz^{2p}w^pD^p - 2^pz^pD^p \\
&+ 3^pw^{3p}C^pa_6^p - 3^pD^pz^pa_6^pw^{2p}\Big)P_{V,1}\Bigg)p \\
&+ \Big(w^{2p}A^p - 2w^{3p}a_4^pz^pA^p + 2w^{2p}z^pB^p - 3^pw^{4p}a_6^pA^p - 2^pw^{2p}A^p\Big)P_{U,0} + \Big(2^pC^pw^p - D^pz^p \\
&+ 2w^{2p}a_4^pz^pC^p - w^pC^p - 2a_4^pz^{2p}w^pD^p - 2^pz^pD^p + 3^pw^{3p}C^pa_6^p - 3^pD^pz^pa_6^pw^{2p}\Big)P_{V,0}
\end{aligned}$$

and $\delta\mu$ is modulo p

$$\begin{aligned}
\delta\mu = \Big(&- B^pw^{2p} - 3(3^p)z^{2p}w^pA^p - 3w^{3p}a_4^pz^pB^p - 2^pw^{3p}a_4^pA^p - 2(2^p)w^{3p}a_4^pz^pB^p \\
&- 2^pw^{3p}B^pa_4^pz^p - 3(3^p)z^{3p}w^pB^p - w^{3p}a_4^pA^p - 3(3^p)a_6^pw^{4p}B^p\Big)PU,0 \\
&+ \Big(- 2(2^p)D^pa_4^pz^pw^p - 2^pw^{2p}a_4^pC^p - 3(3^p)C^pz^{2p} - 2a_4^pz^pw^pD^p - D^p - w^{2p}a_4^pC^p \\
&- 3(3^p)a_6^pD^pw^{2p}\Big)PV,0 \quad + 2\delta(3)z^{3p} + 2\delta(2)w^{2p}a_4^pz^p \\
&+ 2(2^p)w^{2p}\delta(a_4)z^p + 2w^{2p}\delta(a_4)z^p + 2\delta(3)a_6^pw^{3p} + 2(3^p)\delta(a_6)w^{3p} \\
&C_p^{ext}(3a_6w^3 + 3a_6w^3 + 3z^3 + 3z^3 + a_4w^2z + a_4w^2z + w + w2a_4w^2z + 2a_4w^2z)
\end{aligned}$$

Proposition 6 *Let* $\log_{\mathcal{F}_1^{\phi 1}}(\xi)$ *be the formal logarithm of the Frobenius twist of the formal group of the elliptic curve. Then*

$$\log_{\mathcal{F}_1^{\phi 1}}(\zeta) = \zeta \text{ modulo } p^2$$

Proof:

Recall

$$\log_{\mathcal{F}_1^{\phi 1}}(\xi) := \xi + \frac{p\phi(c_1)}{2}\xi^2 + \frac{p^2\phi(c_2)}{3}\xi^3 + \ldots$$

where the c_i are the coefficients of the power series expansion of the invariant differential. We also know that the invariant differential

$$\omega(z) = (1 + 2a_4 z^4 + \ldots)dz$$

and so $c_1 = 0$, $c_2 = 0$, $c_3 = 0$, $c_4 = 2a_4 \ldots$. Hence modulo p^2 the power series $\log_{\mathcal{F}_1^{\phi 1}}(\xi)$ is the identity.

We provide an explicit formula for the residue of $\frac{x^a}{y^b}$ which we will call $\gamma_{a,b}$.

We let $\binom{n}{k}$ denote the binomial coefficient with the convention that $\binom{n}{k} = 0$ if $k > n$.

Proposition 7 *Let a and b be positive integers. Let m and $n \in \{0, 1, 2\}$ be integers such that $a = 3m + n$. Then the residue of $\frac{x^a}{y^b}$ is*

$$\gamma_{a,b} = \begin{cases} 0 & \text{if } b \text{ is even;} \\ \sum_{k=0}^{\infty} \binom{m+k}{3k+2-n} \binom{m-2k-2+n}{\frac{b-1}{2}} (-1)^{m+k-\frac{b-1}{2}} (a_4)^{3k+2-n} (a_6)^{m-2k-2+n-\frac{b-1}{2}} & \text{if } b \text{ is odd.} \end{cases}$$

Using this formula, we define some more notation.

Definition 3 *Define $\mu_{a,b}$ to be the residue class of $\frac{x^a \Upsilon}{y^b}$ where $\Upsilon = C_p^{ext}(y^2 - x^3 - a_4 x - a_6)$.*

Definition 4 *Define $\tau_{a,b}$ to be the residue class of $\frac{x^a \Upsilon^2}{y^b}$ where $\Upsilon = C_p^{ext}(y^2 - x^3 - a_4 x - a_6)$.*

Theorem 8 *The reduction modulo p^2 of f_{jet} the unique, isogeny covariant δ-modular form of weight $\chi_{-p-1,-1}$ and order 1 is*

$$\left[\frac{72^p(2^p-4^p-2(3^p))a_6^p\delta(a_4)}{\Delta^p}+\frac{16^p(-6^p+12^p+2(9^p))a_4^p\delta(a_6)}{\Delta^p}\right]\gamma_{2p,p}$$
$$+\frac{8^p}{\Delta^p}\Big[2^p(1-2^p)4^pa_4^p\mu_{0,p}+(-18^p(1-2^p)+2(27^p))a_6\mu_{p,p}$$
$$+(12^p(1-2^p)-2(18^p)(1+a_6^p))a_4^p\mu_{2p,p}-36^pa_4^pa_6^p\mu_{2p,3p}-2(12^p)a_4^{2p}\mu_{3p,3p}\Big]$$
$$+p\Big(H_0+H_1+H_2+H_3+H_4+H_5+H_6+H_7\Big)$$

where H_0 is

$$\frac{1}{p}\Bigg((1-2^p)\frac{2^{4p}}{\Delta^p}\Big[-\delta(a_4)6^pa_4^p\gamma_{3p,p}$$
$$+\sum_{k=1}^{p-1}\binom{p}{k}(4a_4)^{p-k}\sum_{i=1}^{k}\binom{k}{i}(6a_4)^i(-9a_6)^{k-i}(-\delta(a_4)\gamma_{p+k+i,p}-\delta(a_6)\gamma_{k+i,p}+\mu_{k+i,p})$$
$$+\sum_{k=1}^{p-1}\binom{p}{k}(6a_4)^k(-9a_6)^{p-k}(-\delta(a_4)\gamma_{2p+k,p}-\delta(a_6)\gamma_{p+k,p}+\mu_{p+k,p})\Big]$$
$$-2\frac{2^{3p}(9^p)}{\Delta^p}\Big[-\delta(a_4)2^pa_4^p\gamma_{3p,p}+\sum_{k=1}^{p-1}\binom{p}{k}(2a_4)^k(-3a_6)^{p-k}(-\delta(a_4)\gamma_{p+k,p}+\mu_{k,p}\Big]$$
$$-(3^pa_6^p)\frac{2^{4p}}{\Delta^p}\Big[\sum_{k=1}^{p-1}\binom{p}{k}(4a_4)^{p-k}\sum_{i=1}^{k}\binom{k}{i}(6a_4)^i(-9a_6)^{k-i}\mu_{k+i,3p}$$
$$+\sum_{k=1}^{p-1}\binom{p}{k}(6a_4)^k(-9a_6)^{p-k}\mu_{p+k,3p}\Big]$$
$$-(2a_4^p)\frac{2^{4p}}{\Delta^p}\Big[\sum_{k=1}^{p-1}\binom{p}{k}(4a_4)^{p-k}\sum_{i=1}^{k}\binom{k}{i}(6a_4)^i(-9a_6)^{k-i}\mu_{p+k+i,3p}$$
$$+\sum_{k=1}^{p-1}\binom{p}{k}(6a_4)^k(-9a_6)^{p-k}\mu_{2p+k,3p}\Big]\Bigg)$$

H_1 *is*

$$(\frac{9}{2}\,\delta(a_4)\,{a_4}^p\,2(3^p)\gamma_{3p,p}+\Bigg(-\frac{9}{2}\,{a_4}^p(2(3a_6)^p\mu_{2p,3p}+2(3^p)\mu_{5p,3p}+(2^p+4^p)a_4^p\mu_{3p,3p}+(2-8^p)\mu_{2p,p})$$

$$+(-\frac{3}{2}\,{a_4}^p+9\,{a_6}^p)\,(2(3^p)\mu_{4p,3p}+(2^p+4^p)a_4^p\mu_{2p,3p}+(2-8^p)\mu_{p,p})-{a_4}^{2p}(2(3^p)\mu_{3p,3p}+(2-8^p)\mu_{0,p})\Bigg)$$

$$+\,(-9\,\delta(a_4)\,{a_6}^p+\frac{9}{2}\,{a_4}^p\,\delta(a_6)+\frac{3}{2}\,\delta(a_4)\,{a_4}^p)\,(2(3^p)\mu_{3p,3p}+(2-8^p)\mu_{0,p})$$

$$+((-3\,{a_4}^{2p}+\frac{9}{2}\,{a_6}^p\,{a_4}^p)\,(2(3^p)\mu_{5p,5p}+(2-8^p)\mu_{2p,3p})/\Delta,$$

H_2 *is*

$$\frac{1}{p}\sum_{k=1}^{p-1}\sum_{i=0}^{k}\sum_{j=0}^{p-i}\binom{p}{k}\binom{k}{i}\binom{p-i}{j}(-1)^{k-i}\left(\frac{9(2^4)}{\Delta}\right)^{p-i}(2a_4)^j(-3a_6)^{p-i-j}\Bigg[\Big((-36\,{a_6}^p\,{a_4}^p$$

$$+24\,{a_4}^{2p})\,\mu_{2p+j,3p+2(p-i)}+(36\,{a_6}^p\,{a_4}^p+16\,{a_4}^{3p}-54\,{a_6}^{2p})\,\mu_{p+j,3p+2(p-i)}+24\,{a_6}^p\,{a_4}^{2p}\,\mu_{j,3p+2(p-i)}$$

$$+36\,{a_4}^p\mu_{2p+j,p+2(p-i)}+(-72\,{a_6}^p+12\,{a_4}^p)\,\mu_{p+j,p+2(p-i)}+8\,{a_4}^{2p}\mu_{j,p+2(p-i)}\Big)/\Delta$$

$$+((36\,{a_4}^p\,\delta(a_4)\gamma_{3p+j,p+2(p-i)}+(72\,\delta(a_4)\,{a_6}^p-12\,\delta(a_4)\,{a_4}^p-36\,{a_4}^p\,\delta(a_6))\gamma_{2p+j,p+2(p-i)}$$

$$+\,(28\,{a_4}^{2p}\,\delta(a_4)+72\,\delta(a_6)\,{a_6}^p-12\,{a_4}^p\,\delta(a_6))\,\gamma_{p+j,p+2(p-i)}+(36\,\delta(a_4)\,{a_4}^p\,{a_6}^p$$

$$+\,36\,\delta(a_4)\,{a_4}^p\,a_6-8\,{a_4}^{2p}\,\delta(a_6)-24\,{a_4}^{2p}\,\delta(a_4))\gamma_{j,p+2(p-i)})$$

$$+((-16\,\delta(a_4)\,{a_4}^{3p}-36\,\delta(a_4)\,{a_4}^p\,{a_6}^p+54\,\delta(a_4)\,{a_6}^{2p}-24\,{a_4}^{2p}\,\delta(a_6)+36\,{a_4}^p\,\delta(a_6)\,{a_6}^p)\gamma_{2p+j,3p+2(p-i)}$$

$$+(24\,\delta(a_4)\,{a_4}^{3p}-60\,\delta(a_4)\,{a_6}^p\,{a_4}^{2p}-16\,\delta(a_6)\,{a_4}^{3p}-36\,{a_4}^p\,\delta(a_6)\,{a_6}^p+54\,\delta(a_6)\,{a_6}^{2p})\gamma_{2p+j,3p+2(p-i)})/\Delta\Bigg]$$

H_3 *is*

$$-\frac{1}{p}\sum_{k=1}^{p-1}\binom{p}{k}3^k a_4^{p-k}\Bigg(((2304\,{a_4}^{3p}-8640\,{a_6}^p\,{a_4}^p+10368\,{a_6}^{2p}+1152\,{a_4}^{2p})\,\mu_{2p+2k,p}$$

$$+\,(-576\,{a_6}^p\,{a_4}^{2p}-1920\,{a_4}^{3p})\,\mu_{p+2k,p}$$

$$+(2304\,{a_4}^{3p}+512\,{a_4}^{4p}-10368\,{a_6}^p\,{a_4}^{2p}+10368\,{a_6}^{2p}\,{a_4}^p)\mu_{2k,p})\Big/\Delta^2$$

$$+(((576\,\delta(a_4)\,{a_6}^p\,{a_4}^{2p}-1152\,{a_4}^{2p}\,\delta(a_6)+1920\,\delta(a_4)\,{a_4}^{3p}$$

$$-\,2304\,\delta(a_6)\,{a_4}^{3p}-10368\,\delta(a_6)\,{a_6}^{2p}+8640\,{a_4}^p\,\delta(a_6)\,{a_6}^p)\gamma_{2p+2k,p}$$

$$+(576\,{a_6}^p\,{a_4}^{2p}\,\delta(a_6)-1152\,\delta(a_4)\,{a_4}^{3p}+1792\,\delta(a_4)\,{a_4}^{4p}$$

$$+1728\,\delta(a_4)\,{a_6}^p\,{a_4}^{2p}+1920\,\delta(a_6)\,{a_4}^{3p})\gamma_{p+2k,p}$$

$$(\,-\,2304\,\delta(a_6)\,{a_4}^{3p}+2304\,\delta(a_4)\,{a_4}^{3p}\,a_6$$

$$+\,4608\,\delta(a_4)\,{a_6}^p\,{a_4}^{3p}-10368\,\delta(a_6)\,{a_6}^{2p}\,{a_4}^p+10368\,{a_6}^p\,{a_4}^{2p}\,\delta(a_6)$$

$$-\,3456\,\delta(a_4)\,{a_6}^p\,{a_4}^{2p}+1152\,\delta(a_4)\,{a_4}^{2p}\,a_6-7776\,\delta(a_4)\,{a_6}^{3p}$$

$$-\,3072\,\delta(a_4)\,{a_4}^{4p}-512\,\delta(a_6)\,{a_4}^{4p}-8640\,\delta(a_4)\,{a_4}^p\,{a_6}^{(1+p)}$$

$$+\,10368\,\delta(a_4)\,{a_6}^{2p}\,{a_4}^p+10368\,\delta(a_4)\,{a_6}^{(1+2\,p)})\gamma_{2k,p}))\Big/\Delta^2\Bigg)$$

H_4 *is*

$$
\begin{aligned}
&(\frac{1}{2}(829440\, a_6{}^p\, a_4{}^{4p} - 774144\, a_4{}^{5p} - 1990656\, a_6{}^p\, a_4{}^{(3\,p)} + 12960\, a_6{}^p\, a_4{}^p\, \Delta \\
&- 3456\, a_4{}^{2p}\, \Delta + 3732480\, a_6{}^{2p}\, a_4{}^{2p} - 3359232\, a_6{}^{3p}\, a_4{}^p + 663552\, a_4{}^{4p})\tau_{2p,p} \Big/ \Delta^3 \\
&+\frac{1}{2}(7713792\, a_6{}^{2p}\, a_4{}^{3p} + 3317760\, a_4{}^{5p} - 19440\, a_6{}^{2p}\, \Delta \\
&+ 288\, a_4{}^{3p}\, \Delta + 1327104\, a_6{}^p\, a_4{}^{3p} - 3359232\, a_6{}^{3p}\, a_4{}^p - 24576\, a_4{}^{6p} \\
&+ 12960\, a_6{}^p\, a_4{}^p\, \Delta + 7278336\, a_6{}^{4p} - 2592\, a_4{}^{2p}\, \Delta - 9123840\, a_6{}^p\, a_4{}^{4p} \\
&- 2985984\, a_6{}^{2p}\, a_4{}^{2p})\tau_{p,p} \Big/ \Delta^3 + \frac{1}{2}(-3456\, a_4{}^{3p}\, \Delta - 2875392\, a_4{}^{5p}\, a_6{}^p \\
&- 1327104\, a_4{}^{5p} + 1769472\, a_4{}^{6p} + 8957952\, a_6{}^{3p}\, a_4{}^{2p} \\
&+ 10616832\, a_6{}^p\, a_4{}^{4p} - 20404224\, a_6{}^{2p}\, a_4{}^{3p} + 11232\, a_4{}^{2p}\, a_6{}^p\, \Delta)\tau_{0,p} \Big/ \Delta^3) \\
&+ (\frac{1}{2}(-3317760\, a_6{}^p\, a_4{}^{4p} + 10616832\, a_4{}^{5p}\, a_6{}^p - 2654208\, a_4{}^{6p} \\
&+ 14929920\, a_6{}^{2p}\, a_4{}^{3p} + 11197440\, a_6{}^{4p}\, a_4{}^p - 1152\, a_4{}^{3p}\, \Delta \\
&- 9953280\, a_6{}^{2p}\, a_4{}^{4p} - 1152\, a_4{}^{4p}\, \Delta + 393216\, a_4{}^{(7\,p)} + 3456\, a_4{}^{2p}\, a_6{}^p\, \Delta \\
&- 22394880\, a_6{}^{3p}\, a_4{}^{2p} - 2592\, a_6{}^{2p}\, a_4{}^p\, \Delta)\tau_{2p,3p} \Big/ \Delta^3 \\
&+\frac{1}{2}(-12607488\, a_4{}^{5p}\, a_6{}^p - 5184\, a_4{}^{2p}\, a_6{}^p\, \Delta + 1327104\, a_4{}^{6p} + 2304\, a_4{}^{3p}\, a_6{}^p\, \Delta \\
&+ 28864512\, a_6{}^{2p}\, a_4{}^{4p} - 1769472\, a_4{}^{(7\,p)} - 1990656\, a_6{}^{2p}\, a_4{}^{3p} \\
&- 1536\, a_4{}^{4p}\, \Delta + 3244032\, a_6{}^p\, a_4{}^{6p} - 11664\, a_6{}^{3p}\, \Delta + 15552\, a_6{}^{2p}\, a_4{}^p\, \Delta \\
&+ 8957952\, a_6{}^{3p}\, a_4{}^{2p} - 19408896\, a_6{}^{3p}\, a_4{}^{3p} - 13436928\, a_6{}^{4p}\, a_4{}^p \\
&+ 6718464\, a_6{}^{5p})\tau_{p,3p} \Big/ \Delta^3 + \frac{1}{2}(20901888\, a_6{}^{3p}\, a_4{}^{3p} + 3456\, a_4{}^{4p}\, \Delta \\
&- 9953280\, a_6{}^{2p}\, a_4{}^{4p} - 13436928\, a_6{}^{4p}\, a_4{}^{2p} + 2654208\, a_6{}^{2p}\, a_4{}^{5p} \\
&- 1769472\, a_6{}^p\, a_4{}^{6p} + 18144\, a_6{}^{2p}\, a_4{}^{2p}\, \Delta - 17280\, a_4{}^{3p}\, a_6{}^p\, \Delta \\
&- 1664\, a_4{}^{5p}\, \Delta + 1327104\, a_4{}^{(5\,p)}\, a_6{}^p)\tau_{0,3p} \Big/ \Delta^3) + (\frac{1}{2}(23328\, \Delta\, a_6{}^{3p}\, a_4{}^p \\
&- 31104\, a_6{}^{2p}\, a_4{}^{2p}\, \Delta - 10368\, \Delta\, a_6{}^p\, a_4{}^{4p} + 54\, \Delta^2\, a_4{}^p + 4608\, a_4{}^{5p}\, \Delta \\
&+ 10368\, a_4{}^{3p}\, a_6{}^p\, \Delta)\tau_{2p,5p} \Big/ \Delta^3
\end{aligned}
$$

H_5 *is*

$$
\begin{aligned}
&(\frac{1}{2}(6718464\,\delta(a_4)\,{a_6}^{3p}\,{a_4}^{p} + 5971968\,\delta(a_4)\,{a_6}^{2p}\,{a_4}^{2p} + 18247680\,\delta(a_4)\,{a_6}^{p}\,{a_4}^{4p} \\
&- 15427584\,\delta(a_4)\,{a_6}^{2p}\,{a_4}^{3p} - 5184\,\delta(a_4)\,{a_4}^{3p}\,\Delta + 3584\,{a_4}^{4p}\,\delta(3)\,\Delta - 7464960\,\delta(a_6)\,{a_6}^{2p}\,{a_4}^{2p} \\
&+3981312\,\delta(a_6)\,{a_6}^{p}\,{a_4}^{3p} - 1658880\,\delta(a_6)\,{a_6}^{p}\,{a_4}^{4p} + 6718464\,\delta(a_6)\,{a_6}^{3p}\,{a_4}^{p} \\
&+2880\,\delta(a_4)\,{a_4}^{2p}\,\Delta - 2304\,{a_4}^{3p}\,\delta(3)\,\Delta + 18\,\delta(3)\,{a_4}^{p}\,\Delta^2 + 4608\,\delta(2)\,{a_4}^{4p}\,\Delta \\
&+6912\,{a_4}^{2p}\,\delta(a_6)\,\Delta + 18144\,\delta(a_4)\,{a_6}^{2p}\,\Delta + 3456\,\delta(3)\,{a_4}^{2p}\,{a_6}^{p}\,\Delta + 49152\,\delta(a_4)\,{a_4}^{6p} \\
&-14556672\,\delta(a_4)\,{a_6}^{4p} + 1548288\,\delta(a_6)\,{a_4}^{5p} - 6635520\,\delta(a_4)\,{a_4}^{5p} - 25920\,{a_4}^{p}\,\delta(a_6)\,{a_6}^{p}\,\Delta \\
&-8640\,\delta(a_4)\,{a_6}^{p}\,{a_4}^{p}\,\Delta - 2654208\,\delta(a_4)\,{a_6}^{p}\,{a_4}^{3p} - 1327104\,\delta(a_6)\,{a_4}^{4p})\mu_{2p,p}\Big/\Delta^3 \\
&+\frac{1}{2}(-25214976\,\delta(a_4)\,{a_6}^{p}\,{a_4}^{4p} + 7409664\,{a_4}^{5p}\,\delta(a_4)\,{a_6}^{p} - 24634368\,{a_4}^{2p}\,\delta(a_4)\,{a_6}^{3p} \\
&-15427584\,\delta(a_6)\,{a_6}^{2p}\,{a_4}^{3p} - 576\,\delta(a_6)\,{a_4}^{3p}\,\Delta + 48273408\,\delta(a_4)\,{a_6}^{2p}\,{a_4}^{3p} \\
&+3840\,\delta(a_4)\,{a_4}^{3p}\,\Delta - 9984\,{a_4}^{4p}\,\delta(3)\,\Delta + 5971968\,\delta(a_6)\,{a_6}^{2p}\,{a_4}^{2p} - 2654208\,\delta(a_6)\,{a_6}^{p}\,{a_4}^{3p} \\
&+ 18247680\,\delta(a_6)\,{a_6}^{p}\,{a_4}^{4p} + 6718464\,\delta(a_6)\,{a_6}^{3p}\,{a_4}^{p} + 6\,\delta(3)\,{a_4}^{p}\,\Delta^2 - 6912\,\delta(2)\,{a_4}^{4p}\,\Delta \\
&+5184\,{a_4}^{2p}\,\delta(a_6)\,\Delta + 23328\,\delta(2)\,{a_6}^{3p}\,\Delta + 38880\,\delta(a_6)\,{a_6}^{2p}\,\Delta - 24\,\delta(2)\,{a_4}^{p}\,\Delta^2 \\
&-36\,\delta(3)\,{a_6}^{p}\,\Delta^2 + 36\,\delta(2)\,{a_6}^{p}\,\Delta^2 + 5184\,\delta(3)\,{a_6}^{3p}\,\Delta - 4608\,\delta(3)\,{a_4}^{2p}\,{a_6}^{p}\,\Delta \\
&-5087232\,\delta(a_4)\,{a_4}^{6p} + 49152\,\delta(a_6)\,{a_4}^{6p} + 4608\,{a_4}^{2p}\,\delta(a_4)\,{a_6}^{p}\,\Delta + 12672\,{a_4}^{3p}\,\delta(3)\,{a_6}^{p}\,\Delta \\
&- 14556672\,\delta(a_6)\,{a_6}^{4p} - 6635520\,\delta(a_6)\,{a_4}^{5p} + 10368\,\delta(2)\,{a_4}^{3p}\,{a_6}^{p}\,\Delta + 3981312\,\delta(a_4)\,{a_4}^{5p} \\
&-15552\,\delta(2)\,{a_6}^{2p}\,{a_4}^{p}\,\Delta + 3456\,\delta(3)\,{a_6}^{2p}\,{a_4}^{p}\,\Delta - 25920\,{a_4}^{p}\,\delta(a_6)\,{a_6}^{p}\,\Delta)\mu_{p,p}\Big/\Delta^3 \\
&+ \frac{1}{2}(6635520\,\delta(a_4)\,{a_6}^{p}\,{a_4}^{4p} - 22394880\,{a_4}^{p}\,\delta(a_4)\,{a_6}^{4p} + 1658880\,{a_4}^{4p}\,\delta(a_4)\,{a_6}^{(1+p)} \\
&- 21233664\,{a_4}^{5p}\,\delta(a_4)\,{a_6}^{p} + 44789760\,{a_4}^{2p}\,\delta(a_4)\,{a_6}^{3p} + 40808448\,\delta(a_6)\,{a_6}^{2p}\,{a_4}^{3p} \\
&+6912\,\delta(a_6)\,{a_4}^{3p}\,\Delta + 2\,\delta(2)\,{a_4}^{2p}\,\Delta^2 - 29859840\,\delta(a_4)\,{a_6}^{2p}\,{a_4}^{3p} + 5750784\,{a_4}^{5p}\,\delta(a_6)\,{a_6}^{p} \\
&+1280\,{a_4}^{4p}\,\delta(a_4)\,\Delta + 19906560\,{a_4}^{4p}\,\delta(a_4)\,{a_6}^{2p} - 1548288\,{a_4}^{5p}\,\delta(a_4)\,a_6 + 54\,{a_4}^{p}\,\delta(a_4)\,\Delta^2 \\
&- 6718464\,{a_4}^{p}\,\delta(a_4)\,{a_6}^{(3\,p+1)} - 2304\,\delta(a_4)\,{a_4}^{3p}\,\Delta - 17915904\,{a_4}^{2p}\,\delta(a_6)\,{a_6}^{3p} \\
&+4608\,{a_4}^{4p}\,\delta(3)\,\Delta - 14\,\delta(3)\,{a_4}^{2p}\,\Delta^2 + 7464960\,{a_4}^{2p}\,\delta(a_4)\,{a_6}^{(1+2\,p)} + 1327104\,\delta(a_4)\,{a_4}^{4p}\,a_6 \\
&- 21233664\,\delta(a_6)\,{a_6}^{p}\,{a_4}^{4p} - 3981312\,\delta(a_4)\,{a_4}^{3p}\,{a_6}^{(1+p)} + 5308416\,\delta(a_4)\,{a_4}^{6p} \\
&-6912\,{a_4}^{2p}\,\delta(a_4)\,a_6\,\Delta - 2048\,{a_4}^{5p}\,\delta(3)\,\Delta + 23040\,\delta(3)\,{a_6}^{2p}\,{a_4}^{2p}\,\Delta - 22464\,{a_4}^{2p}\,\delta(a_6)\,{a_6}^{p}\,\Delta \\
&-3538944\,\delta(a_6)\,{a_4}^{6p} - 786432\,{a_4}^{(7\,p)}\,\delta(a_4) + 13824\,{a_4}^{2p}\,\delta(a_4)\,{a_6}^{p}\,\Delta - 26880\,{a_4}^{3p}\,\delta(3)\,{a_6}^{p}\,\Delta \\
&- 15552\,{a_4}^{p}\,\delta(a_4)\,{a_6}^{2p}\,\Delta + 2654208\,\delta(a_6)\,{a_4}^{5p} + 25920\,{a_4}^{p}\,\delta(a_4)\,{a_6}^{(1+p)}\,\Delta \\
&- 6912\,\delta(2)\,{a_4}^{3p}\,{a_6}^{p}\,\Delta)\mu_{0,p}\Big/\Delta^3)
\end{aligned}
$$

H_6 *is*

$$(\frac{1}{2}(26873856\, {a_4}^{p}\, \delta(a_4)\, {a_6}^{4p} + 25214976\, {a_4}^{5p}\, \delta(a_4)\, {a_6}^{p}$$
$$-17915904\, {a_4}^{2p}\, \delta(a_4)\, {a_6}^{3p} - 29859840\, \delta(a_6)\, {a_6}^{2p}\, {a_4}^{3p} + 2304\, \delta(a_6)\, {a_4}^{3p}\, \Delta + 6\, \delta(2)\, {a_4}^{2p}\, \Delta^2$$
$$+3981312\, \delta(a_4)\, {a_6}^{2p}\, {a_4}^{3p} - 21233664\, {a_4}^{5p}\, \delta(a_6)\, {a_6}^{p} - 3072\, {a_4}^{4p}\, \delta(a_4)\, \Delta$$
$$-57729024\, {a_4}^{4p}\, \delta(a_4)\, {a_6}^{2p} - 30\, {a_4}^{p}\, \delta(a_4)\, \Delta^2 + 44789760\, {a_4}^{2p}\, \delta(a_6)\, {a_6}^{3p} + 6\, \delta(3)\, {a_4}^{2p}\, \Delta^2$$
$$+6635520\, \delta(a_6)\, {a_6}^{p}\, {a_4}^{4p} + 54\, {a_4}^{p}\, \delta(a_6)\, \Delta^2 - 6488064\, \delta(a_4)\, {a_6}^{p}\, {a_4}^{6p}$$
$$+19906560\, {a_4}^{4p}\, \delta(a_6)\, {a_6}^{2p} + 38817792\, \delta(a_4)\, {a_6}^{3p}\, {a_4}^{3p} + 2304\, {a_4}^{4p}\, \delta(a_6)\, \Delta - 36\, \delta(a_4)\, {a_6}^{p}\, \Delta^2$$
$$+7776\, \delta(a_4)\, {a_6}^{3p}\, \Delta - 22394880\, {a_4}^{p}\, \delta(a_6)\, {a_6}^{4p} - 2654208\, \delta(a_4)\, {a_4}^{6p} + 6144\, {a_4}^{5p}\, \delta(3)\, \Delta$$
$$-34560\, \delta(3)\, {a_6}^{2p}\, {a_4}^{2p}\, \Delta - 6912\, {a_4}^{2p}\, \delta(a_6)\, {a_6}^{p}\, \Delta + 5308416\, \delta(a_6)\, {a_4}^{6p} + 3538944\, {a_4}^{(7\,p)}\, \delta(a_4)$$
$$+3456\, {a_4}^{2p}\, \delta(a_4)\, {a_6}^{p}\, \Delta + 11520\, {a_4}^{3p}\, \delta(3)\, {a_6}^{p}\, \Delta - 10368\, {a_4}^{p}\, \delta(a_4)\, {a_6}^{2p}\, \Delta - 13436928\, \delta(a_4)\, {a_6}^{5p}$$
$$-786432\, {a_4}^{(7\,p)}\, \delta(a_6) - 36\, \delta(2)\, {a_4}^{p}\, {a_6}^{p}\, \Delta^2 + 18\, \delta(3)\, {a_6}^{p}\, {a_4}^{p}\, \Delta^2 + 5184\, {a_4}^{p}\, \delta(a_6)\, {a_6}^{2p}\, \Delta$$
$$+4608\, \delta(a_4)\, {a_4}^{3p}\, {a_6}^{p}\, \Delta - 12288\, {a_4}^{4p}\, \delta(3)\, {a_6}^{p}\, \Delta + 25920\, {a_6}^{3p}\, \delta(3)\, {a_4}^{p}\, \Delta)\mu_{2p,3p} \Big/ \Delta^3$$

and H_7 *is*

$$(-9\, {a_4}^{p}\, \delta(a_6)\, \Delta^2\, \delta(3) + 12\, \delta(a_4)\, \delta(2)\, {a_4}^{p}\, \Delta^2 - 3\, \delta(a_4)\, \delta(3)\, {a_4}^{p}\, \Delta^2 - 18\, \delta(a_4)\, \delta(2)\, {a_6}^{p}\, \Delta^2$$
$$+ 4320\, \delta(a_4)\, {a_4}^{p}\, \delta(a_6)\, {a_6}^{p}\, \Delta + 18\, \delta(a_4)\, \delta(3)\, {a_6}^{p}\, \Delta^2 + 6480\, {a_4}^{p}\, \delta(a_6)^2\, {a_6}^{p}\, \Delta$$
$$- 387072\, {a_4}^{5p}\, \delta(a_6)^2 - 5184\, \delta(a_4)\, \delta(2)\, {a_4}^{3p}\, {a_6}^{p}\, \Delta$$
$$- 1728\, {a_4}^{2p}\, \delta(a_6)\, \Delta\, \delta(3)\, {a_6}^{p} + 7776\, \delta(a_4)\, \delta(2)\, {a_6}^{2p}\, {a_4}^{p}\, \Delta$$
$$+ 1152\, {a_4}^{3p}\, \delta(a_6)\, \Delta\, \delta(3) - 3359232\, \delta(a_4)\, {a_4}^{p}\, \delta(a_6)\, {a_6}^{3p}$$
$$- 11664\, \delta(a_4)\, \delta(2)\, {a_6}^{3p}\, \Delta - 2592\, \delta(a_4)\, \delta(3)\, {a_6}^{3p}\, \Delta - 1792\, {a_4}^{4p}\, \delta(a_6)\, \Delta\, \delta(3)$$
$$- 2304\, {a_4}^{4p}\, \delta(a_6)\, \Delta\, \delta(2) + 7713792\, \delta(a_4)\, \delta(a_6)\, {a_6}^{2p}\, {a_4}^{3p}$$
$$+ 2592\, \delta(a_4)\, \delta(a_6)\, {a_4}^{(3\,p)}\, \Delta - 9072\, \delta(a_4)\, \delta(a_6)\, {a_6}^{2p}\, \Delta$$
$$- 2985984\, \delta(a_4)\, {a_4}^{2p}\, \delta(a_6)\, {a_6}^{2p} - 9123840\, \delta(a_4)\, {a_4}^{(4\,p)}\, \delta(a_6)\, {a_6}^{p}$$
$$+ 331776\, {a_4}^{4p}\, \delta(a_6)^2 - 1440\, {a_4}^{2p}\, \delta(a_4)^2\, {a_6}^{p}\, \Delta - 6336\, \delta(a_4)\, {a_4}^{3p}\, \delta(3)\, {a_6}^{p}\, \Delta$$
$$+ 4992\, \delta(a_4)\, {a_4}^{4p}\, \delta(3)\, \Delta - 1440\, \delta(a_4)\, {a_4}^{2p}\, \delta(a_6)\, \Delta - 1679616\, {a_4}^{p}\, \delta(a_6)^2\, {a_6}^{3p}$$
$$+ 6158592\, {a_4}^{2p}\, \delta(a_4)^2\, {a_6}^{3p} - 1852416\, {a_4}^{5p}\, \delta(a_4)^2\, {a_6}^{p} + 3456\, \delta(a_4)\, \delta(2)\, {a_4}^{4p}\, \Delta$$
$$-1728\, \delta(a_4)\, \delta(3)\, {a_6}^{2p}\, {a_4}^{p}\, \Delta + 2304\, \delta(a_4)\, \delta(3)\, {a_4}^{2p}\, {a_6}^{p}\, \Delta + 1327104\, \delta(a_4)\, {a_4}^{3p}\, \delta(a_6)\, {a_6}^{p}$$
$$- 995328\, {a_4}^{3p}\, \delta(a_6)^2\, {a_6}^{p} + 1271808\, {a_4}^{6p}\, \delta(a_4)^2 - 995328\, {a_4}^{5p}\, \delta(a_4)^2$$
$$- 24576\, \delta(a_4)\, \delta(a_6)\, {a_4}^{(6\,p)} + 3317760\, \delta(a_4)\, {a_4}^{5p}\, \delta(a_6)$$
$$- 12068352\, {a_4}^{3p}\, \delta(a_4)^2\, {a_6}^{2p} + 7278336\, \delta(a_4)\, \delta(a_6)\, {a_6}^{4p}$$
$$+ 6303744\, {a_4}^{4p}\, \delta(a_4)^2\, {a_6}^{p} - 1920\, {a_4}^{3p}\, \delta(a_4)^2\, \Delta - 1728\, {a_4}^{2p}\, \delta(a_6)^2\, \Delta$$
$$+ 1866240\, {a_4}^{2p}\, \delta(a_6)^2\, {a_6}^{2p} + 414720\, {a_4}^{4p}\, \delta(a_6)^2\, {a_6}^{p})\gamma_{2p,p} \Big/ (\Delta^3)$$